CSC 7426 : Introduction to Software and Data Engineering

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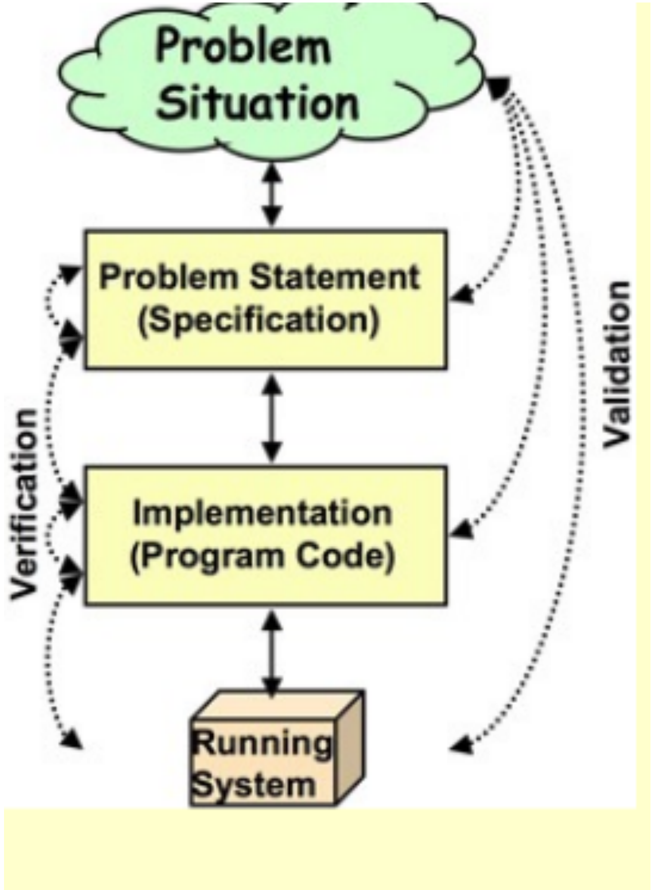
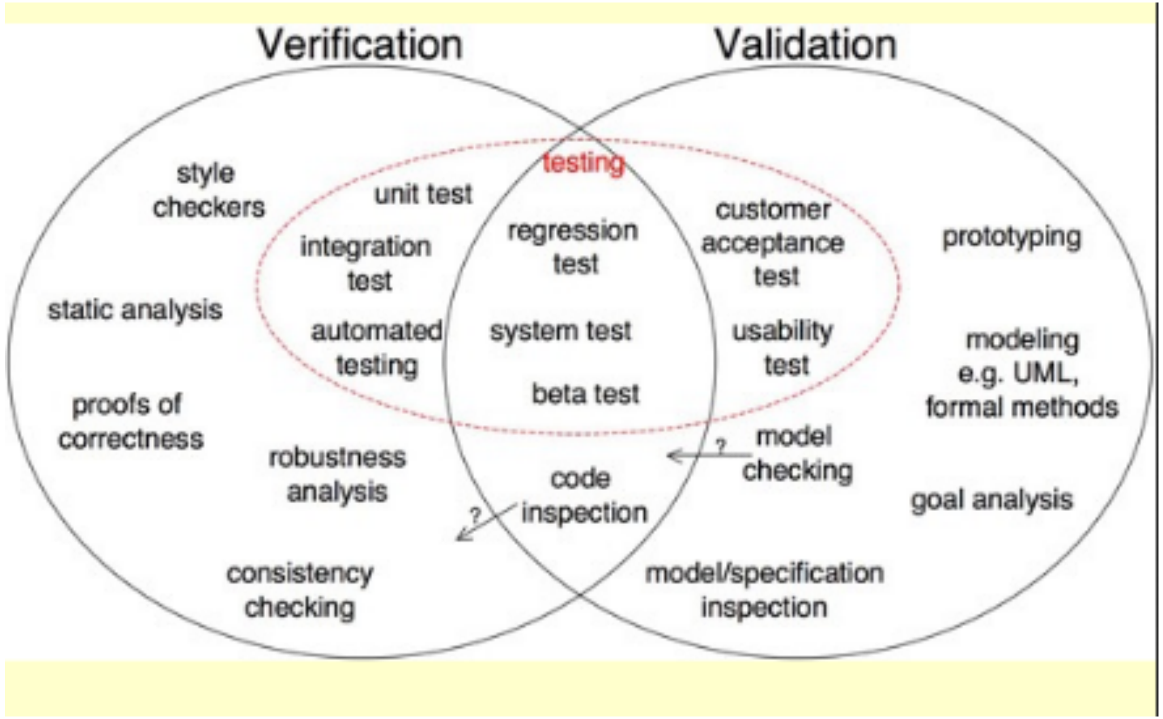
Formal Modelling - Models, Languages and Methods

L5-FormalModelling.pdf

Engineers SOLVE PROBLEMS


Problem Situation
Problem Statement (Specification)
Implementation (Program Code)
Running System
Verification
Validation

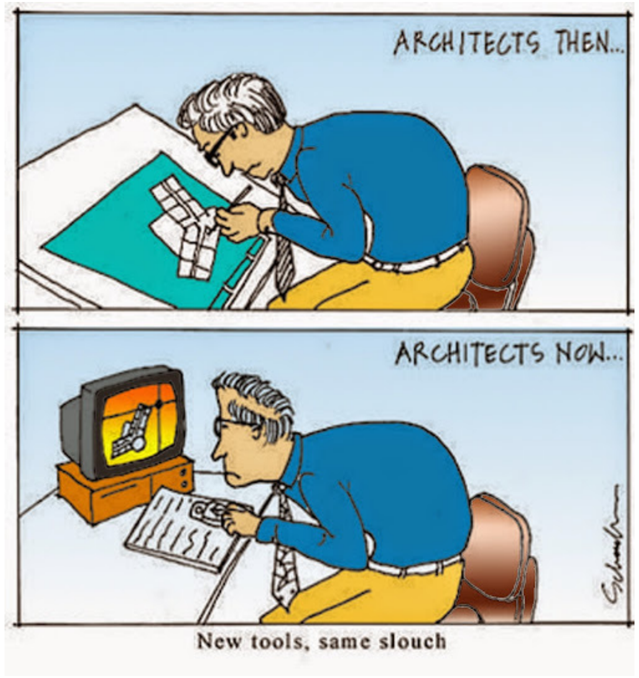


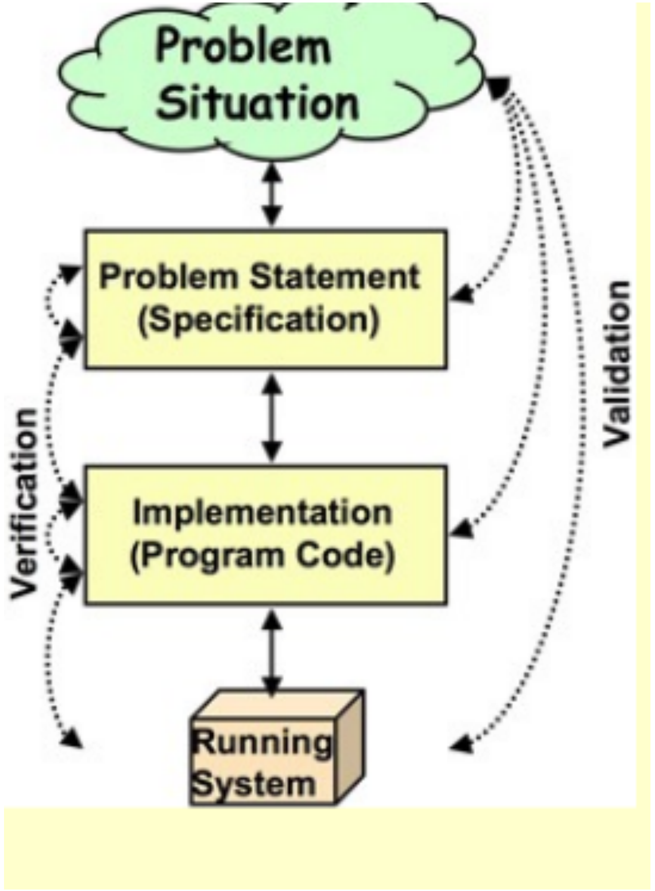
and CHECK (proposed) SOLUTIONS

Engineers work with models (of problems and solutions)

Engineering is based on science – **scientists** (try to) **build models** of things in the real world, **engineers** (try to) build **things** in the real world from models

Architects build models of problems and solutions – they are engineers and scientists

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Build a Requirements Model

Build a Design Model

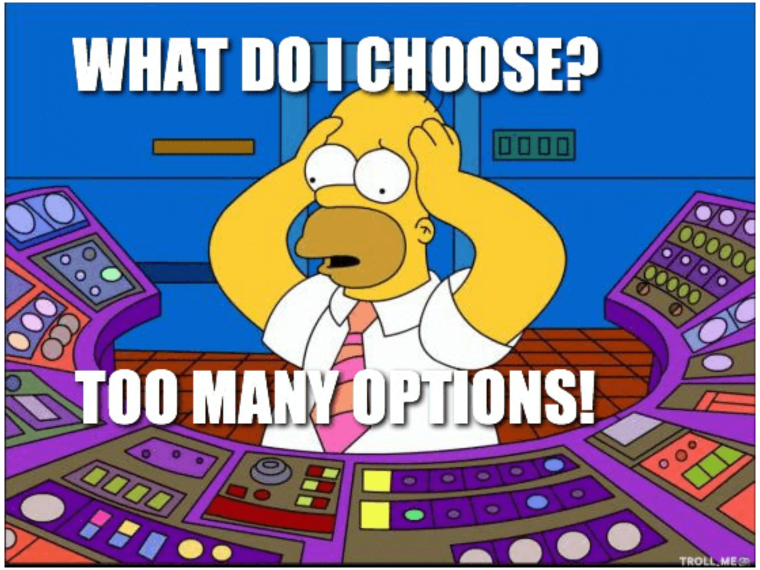
Build an Implementation Model

Build Test Model

What is a good model ?

What is a good modelling language ?

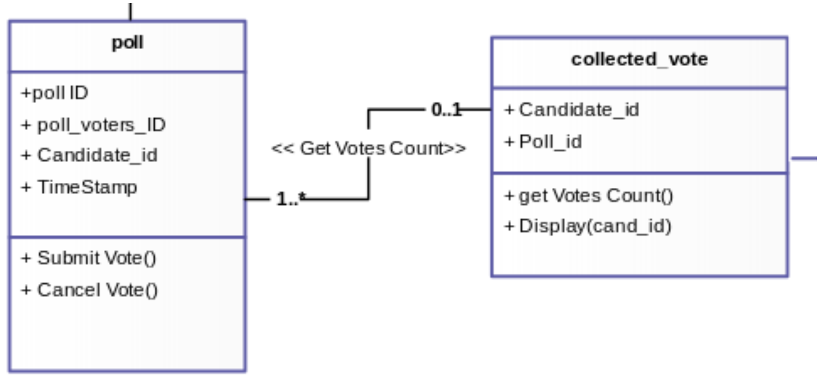
What is a good modelling method ?



Natural Language

An on-line voting system

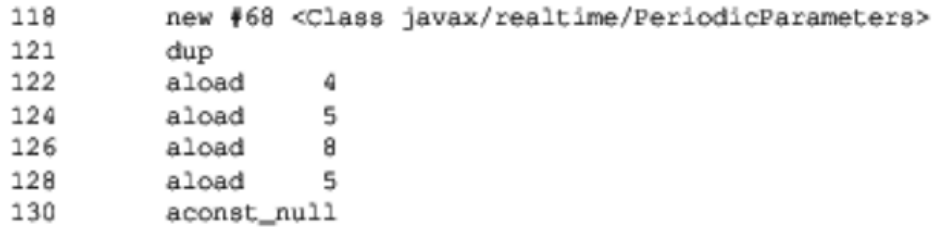
UML



Java

public class Poll { ...}

Java Byte Code



```
118         new #68 <Class javax/realtime/PeriodicParameters>
121         dup
122         aload     4
124         aload     5
126         aload     8
128         aload     5
130         aconst_null
```

Machine Code

11000001010001110100011O

Building a Model

Modelling Method - any technique concerned with the construction and/or analysis of mathematical models which aid the development of computer/information systems

Some *toy* modelling languages will help us explore the fundamental concepts

We will not be using UML/Java but the lessons are the same !!

Typographical Re-write Systems (TRS)

A TRS is a formal system based on the ability to generate a set of strings following a simple set of syntactic rules.

Each rule is calculable --- the generation of a new string from an old string by application of a rule always terminates

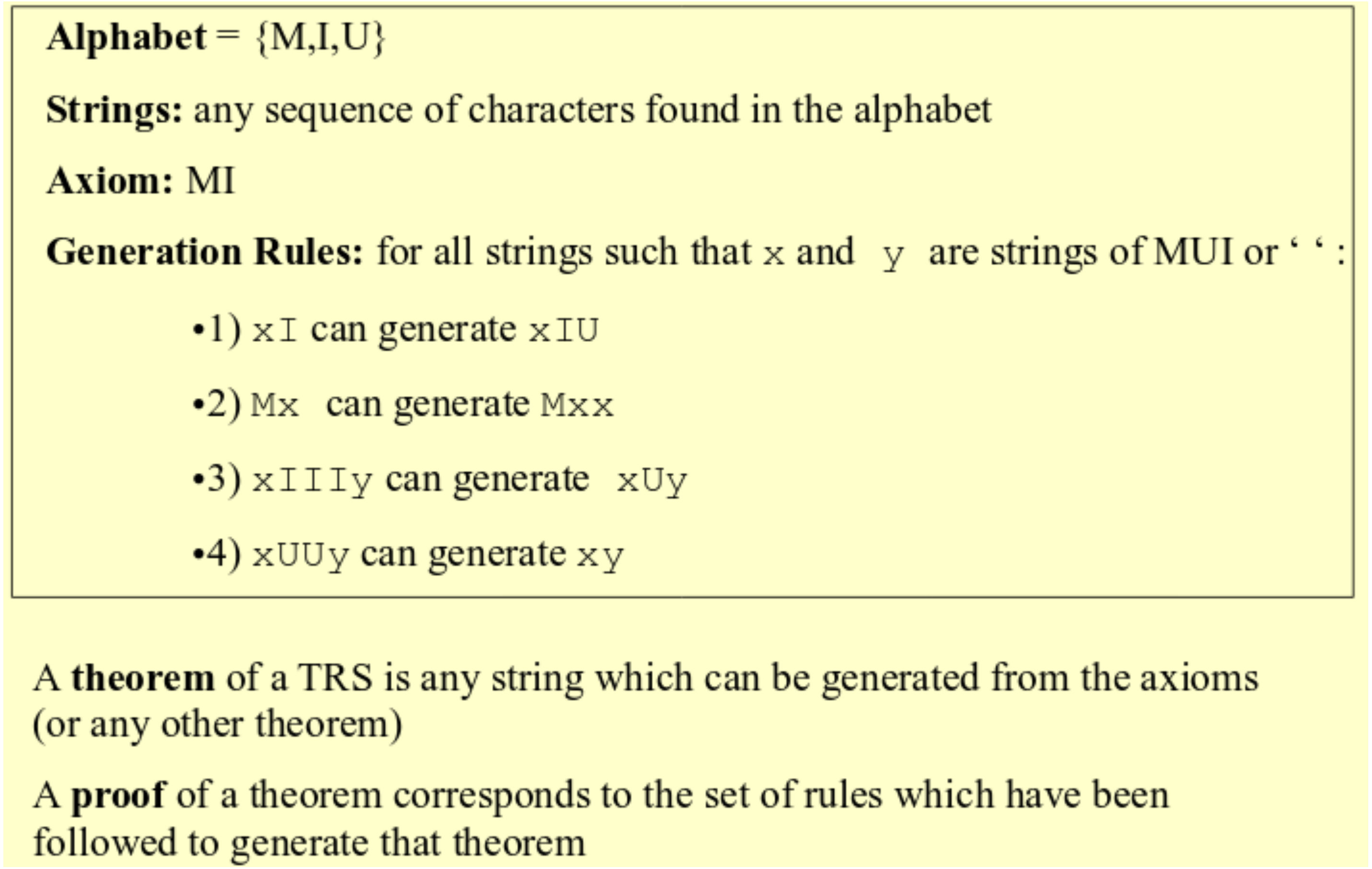
A TRS may produce an infinite number of strings

TRSs can be as powerful as any computing machine

TRSs are simple to implement (simulate)

Case Study 1 --- The MUI TRS

Thanks to Hofstadter - https://en.wikipedia.org/wiki/Gödel,_Escher,_Bach



Alphabet = {M,I,U}

Strings: any sequence of characters found in the alphabet

Axiom: MI

Generation Rules: for all strings such that `x` and `y` are strings of MUI or ‘ ‘ :

- 1) `xI` can generate `xIU`
- 2) `Mx` can generate `Mxx`
- 3) `xIIIy` can generate `xUy`
- 4) `xUUy` can generate `xy`

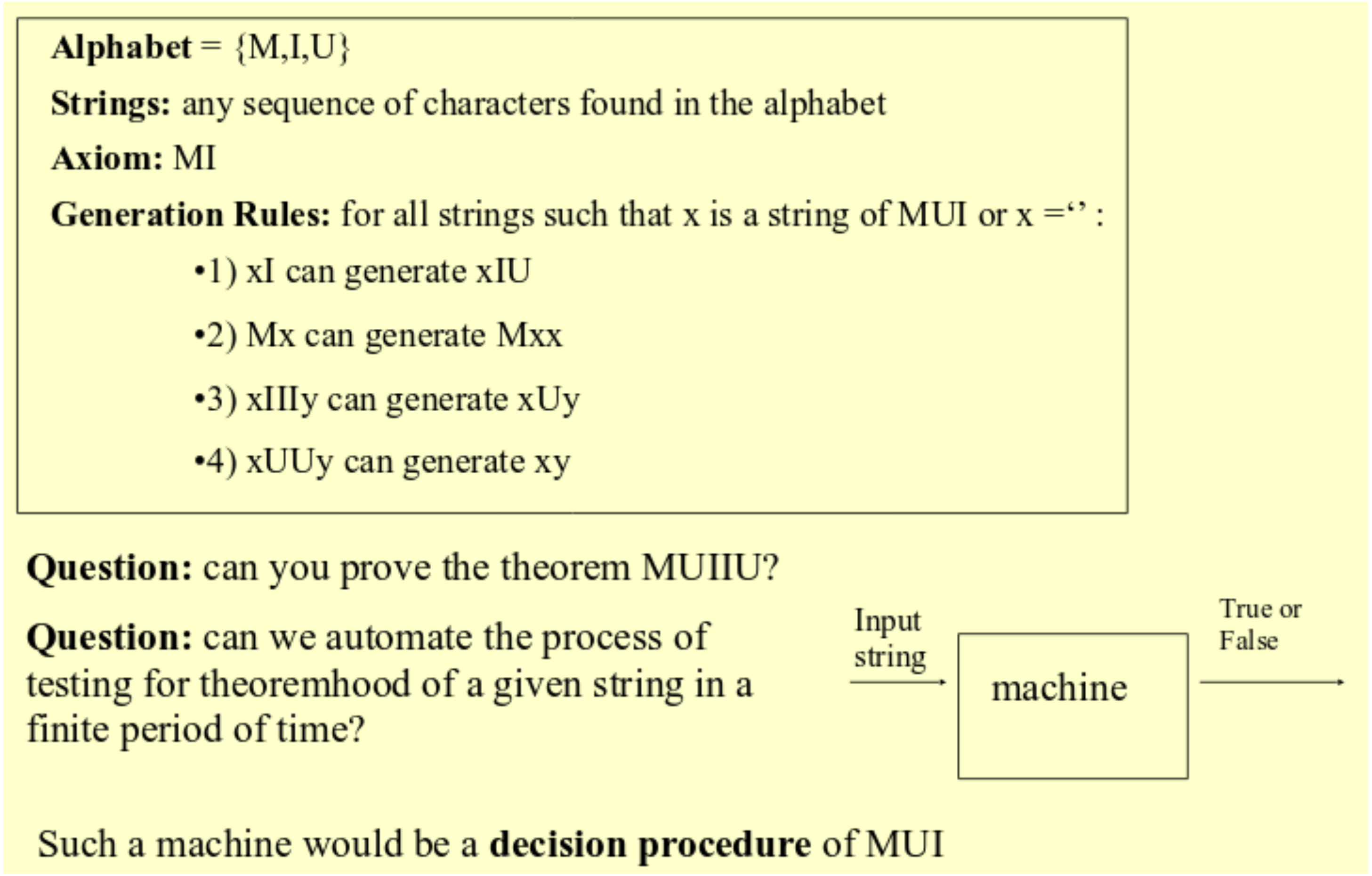
Modelling Language

A **theorem** of a TRS is any string which can be generated from the axioms (or any other theorem)

A **proof** of a theorem corresponds to the set of rules which have been followed to generate that theorem

Models

Case Study 1 --- The MUI TRS



Alphabet = {M,I,U}

Strings: any sequence of characters found in the alphabet

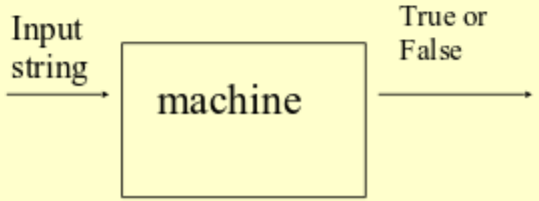
Axiom: MI

Generation Rules: for all strings such that x is a string of MUI or x ='' :

- 1) xI can generate xIU
- 2) Mx can generate Mxx
- 3) xIIIy can generate xUy
- 4) xUUy can generate xy

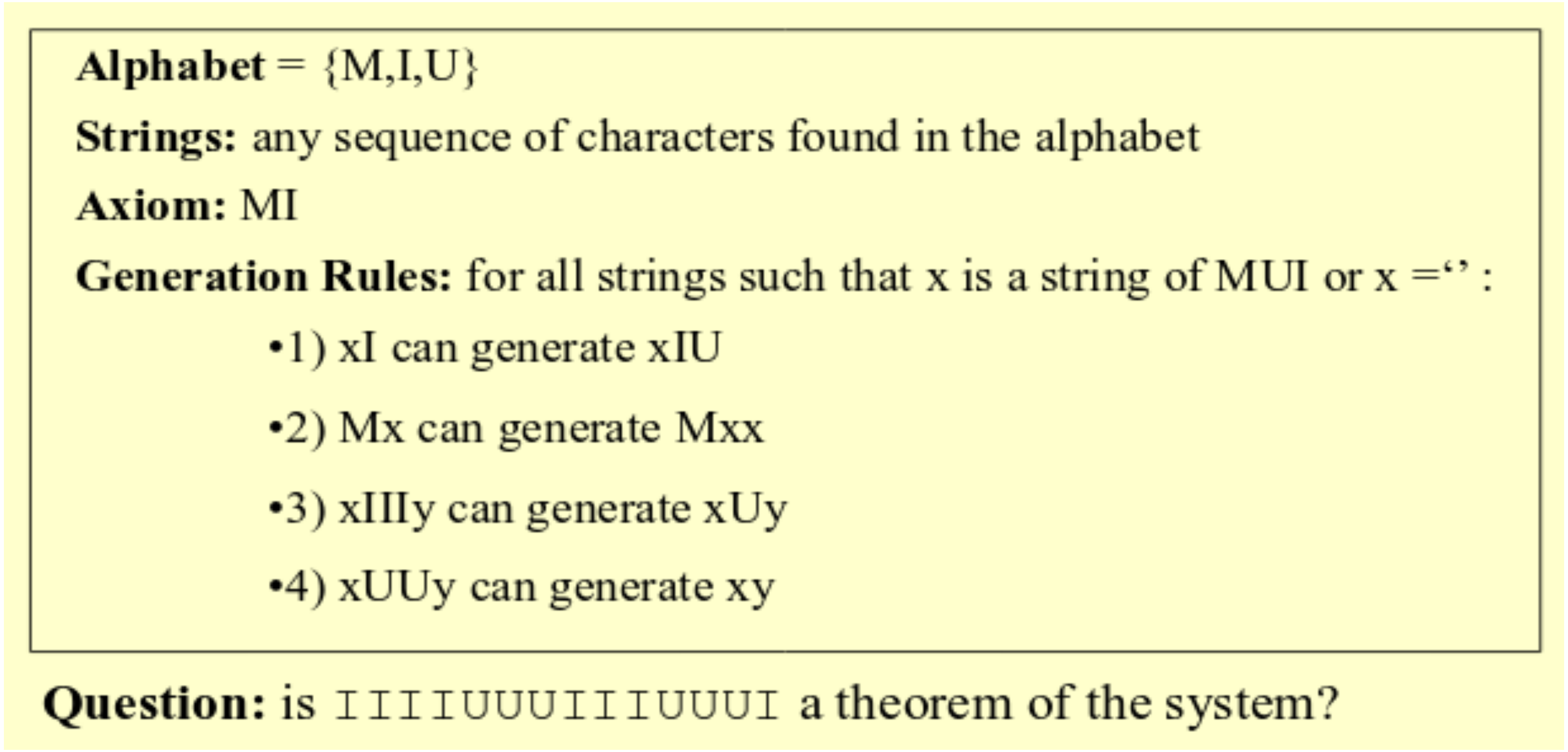
Question: can you prove the theorem MUIIU?

Question: can we automate the process of testing for theoremhood of a given string in a finite period of time?



Such a machine would be a **decision procedure** of MUI

Case Study 1 --- The MUI TRS



Alphabet = {M,I,U}

Strings: any sequence of characters found in the alphabet

Axiom: MI

Generation Rules: for all strings such that x is a string of MUI or x =‘’ :

- •1) xI can generate xIU
- •2) Mx can generate Mxx
- •3) xIIIy can generate xUy
- •4) xUUy can generate xy

Question: is `IIIIUUUIIIUUUI` a theorem of the system?

Question: before we move on ... is MU a theorem of MUI ?

Case Study 2 --- The pq- TRS

Alphabet = {p,q,-}

Axiom: for any such x such that x is a possibly empty sequence of '-'s,

`xp-qx-` is an axiom

Generation Rules: for any x,y,z which are possibly empty sequences of '-'s,
if `xpyqz` is a theorem then `xpy-qz-` is a theorem

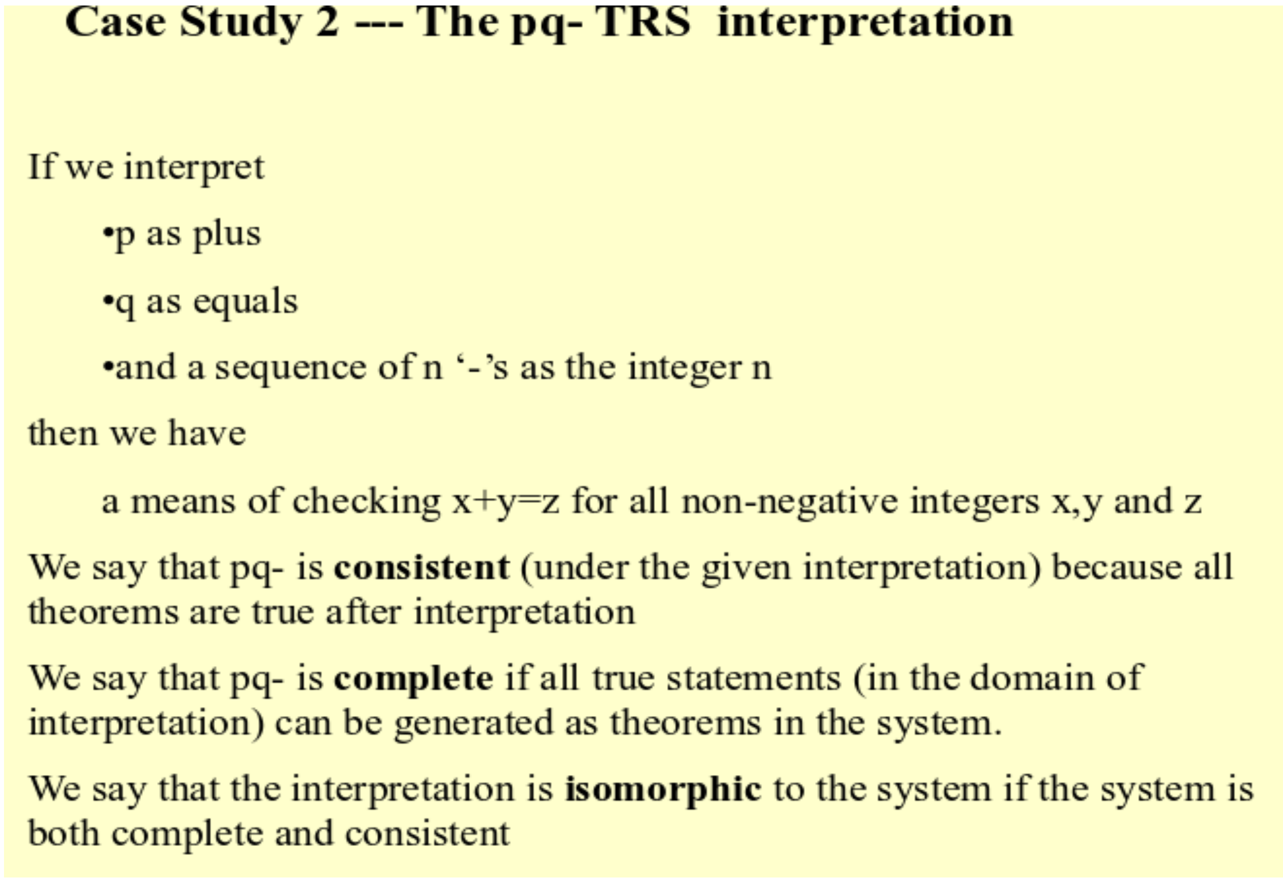
Question: is there a decision procedure for this formal system?

Hint: all re-write rules lengthen the string so …?

Alphabet = {p,q,-}

Axiom:
for any such x such that x is a possibly empty sequence of '-'s,
xp-qx- is an axiom

Generation Rules:
for any x,y,z which are possibly empty sequences of '-'s,
if xpyqz is a theorem
then xpy-qz- is a theorem



Case Study 2 --- The pq- TRS interpretation

If we interpret

- p as plus
- q as equals
- and a sequence of n '-'s as the integer n

then we have

a means of checking x+y=z for all non-negative integers x,y and z

We say that pq- is **consistent** (under the given interpretation) because all theorems are true after interpretation

We say that pq- is **complete** if all true statements (in the domain of interpretation) can be generated as theorems in the system.

We say that the interpretation is **isomorphic** to the system if the system is both complete and consistent

Modellers strive for consistency and completeness

Case Study 2 --- The pq- TRS extension

The pq- system is isomorphic to a very limited domain of interpretation (but maybe that is all that is required!)

Normally, to widen a domain we can

add an axiom

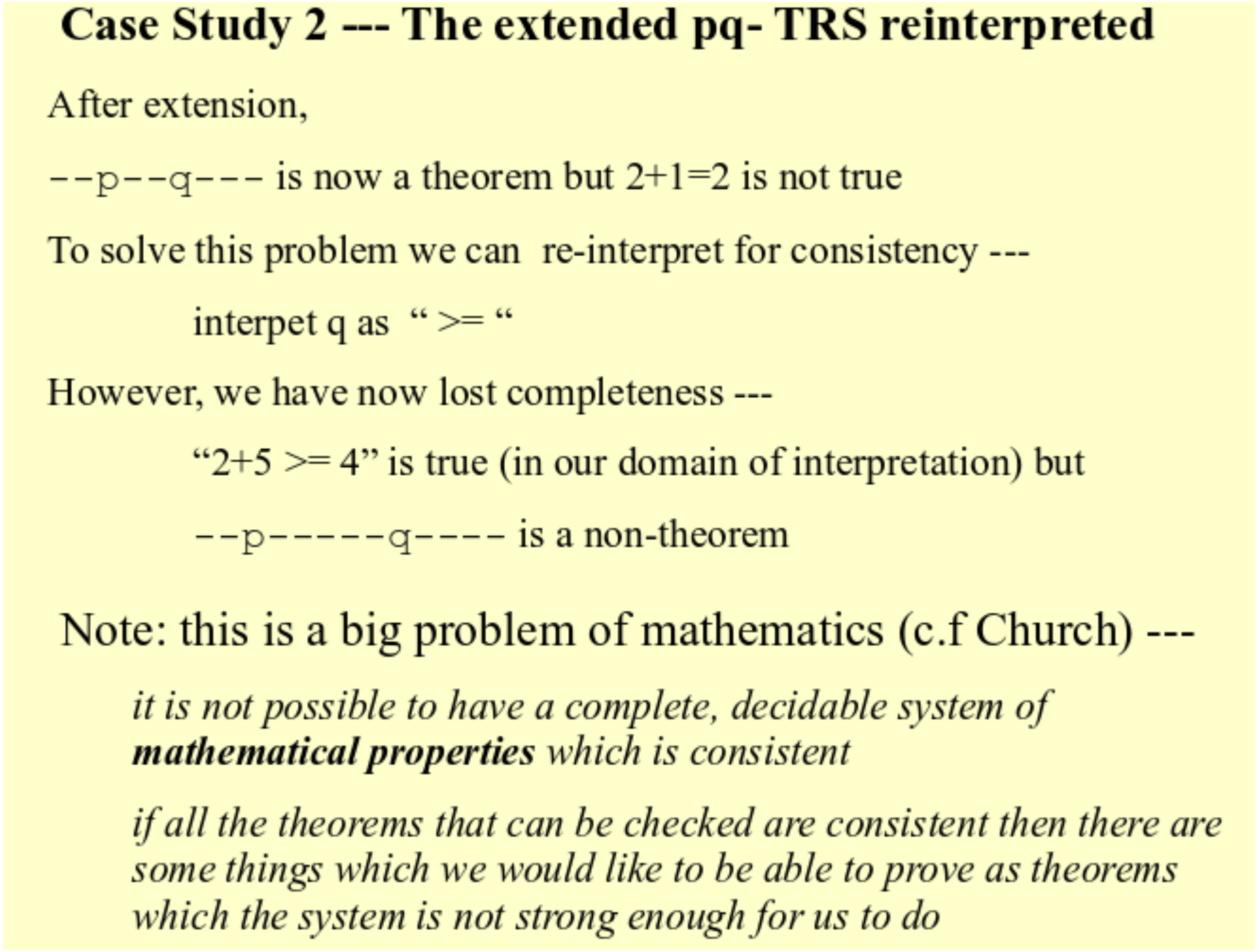
add a generating rule

For example, what happens if we add the axiom:

```
xp-qx.
```

Using this, we can generate many new theorems!

Question: with this new axiom what about completeness and consistency?



Case Study 2 --- The extended pq- TRS reinterpreted

After extension,

`--p--q---` is now a theorem but 2+1=2 is not true

To solve this problem we can re-interpret for consistency ---

interpet q as " >= "

However, we have now lost completeness ---

"2+5 >= 4" is true (in our domain of interpretation) but

`--p-----q----` is a non-theorem

Note: this is a big problem of mathematics (c.f Church) ---

it is not possible to have a complete, decidable system of ***mathematical properties*** *which is consistent*

if all the theorems that can be checked are consistent then there are some things which we would like to be able to prove as theorems which the system is not strong enough for us to do

Question : Impact on Requirements Modelling ?

Case Study 3 --- A tq- TRS

Question:

- can you define a TRS for modelling the multiplication of two integers
- can you show that it is complete and consistent

Interpretation:

- t as times
- q as equals
- sequences of '-'s as integers

Problem -

Define a TRS that can decide if a natural number is composite

Define a TRS that can decide if a natural number is prime

From TRSs to Abstract Data Types (ADTs)

ADTs are a very powerful specification technique which exist in many forms (languages).

These languages are often given operational semantics in a way similar to TRSs (in fact, they are *pretty much equivalent*)

Most ADTs have the following parts ---

- A type which is made up from sorts
- Sorts which are made up of equivalent sets
- Equivalent sets which are made up of expressions

For example, the integer type could be made up of

- sorts integer and boolean
- 1 equivalence set of the integer sort could be {3, 1+2, 2+1, 1+1+1}
- 1 equivalence set of the boolean sort could be {3=3, 1=1, not(false)}

Often used to model requirements

Case Study 4: A simple ADT specification

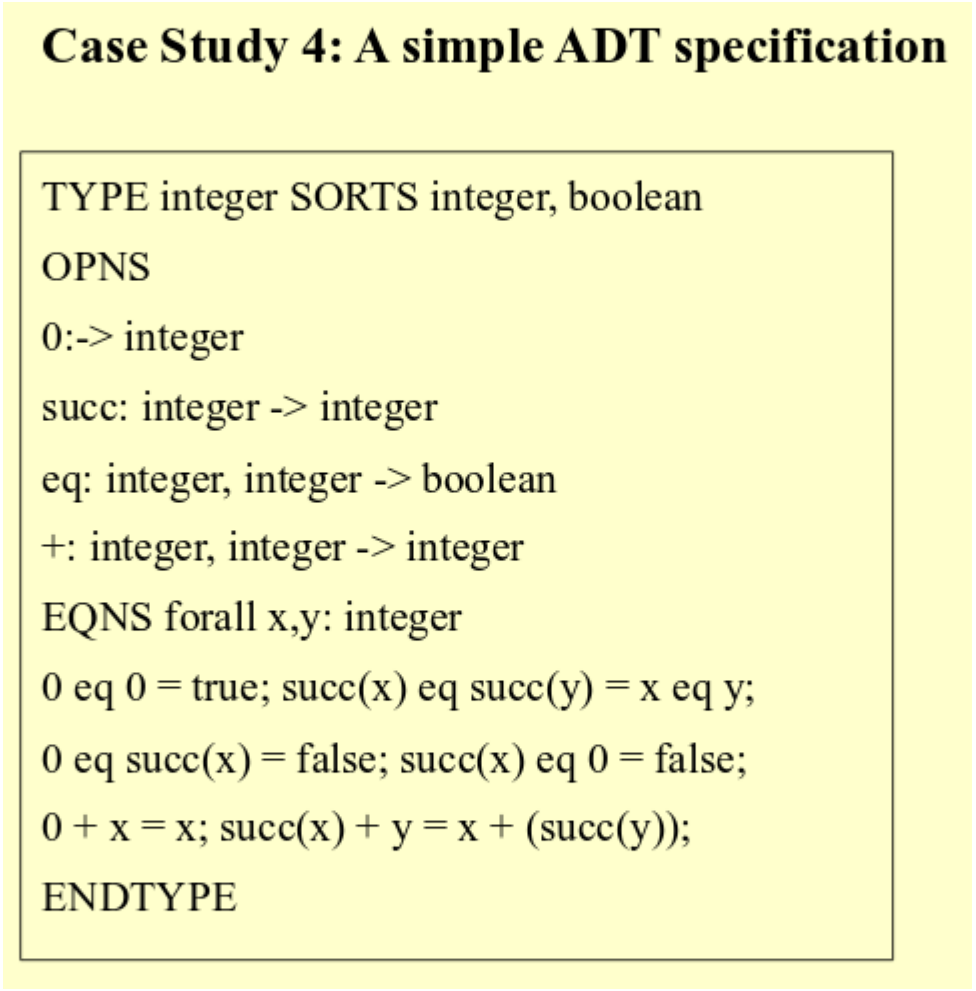
```
TYPE integer SORTS integer, boolean
OPNS
0:-> integer
succ: integer -> integer
eq: integer, integer -> boolean
+: integer, integer -> integer
EQNS forall x,y: integer
0 eq 0 = true; succ(x) eq succ(y) = x eq y;
0 eq succ(x) = false; succ(x) eq 0 = false;
0 + x = x; succ(x) + y = x + (succ(y));
ENDTYPE
```

Question: how do we show, for example ---

- 1+2 = 3,
- 3+2 = 4+1,
- 2+2 != 3+2

Question:

Extend the model to include multiplication



Case Study 4: A simple ADT specification

```
TYPE integer SORTS integer, boolean
OPNS
0:-> integer
succ: integer -> integer
eq: integer, integer -> boolean
+: integer, integer -> integer
EQNS forall x,y: integer
0 eq 0 = true; succ(x) eq succ(y) = x eq y;
0 eq succ(x) = false; succ(x) eq 0 = false;
0 + x = x; succ(x) + y = x + (succ(y));
ENDTYPE
```

Important properties

Redundancy

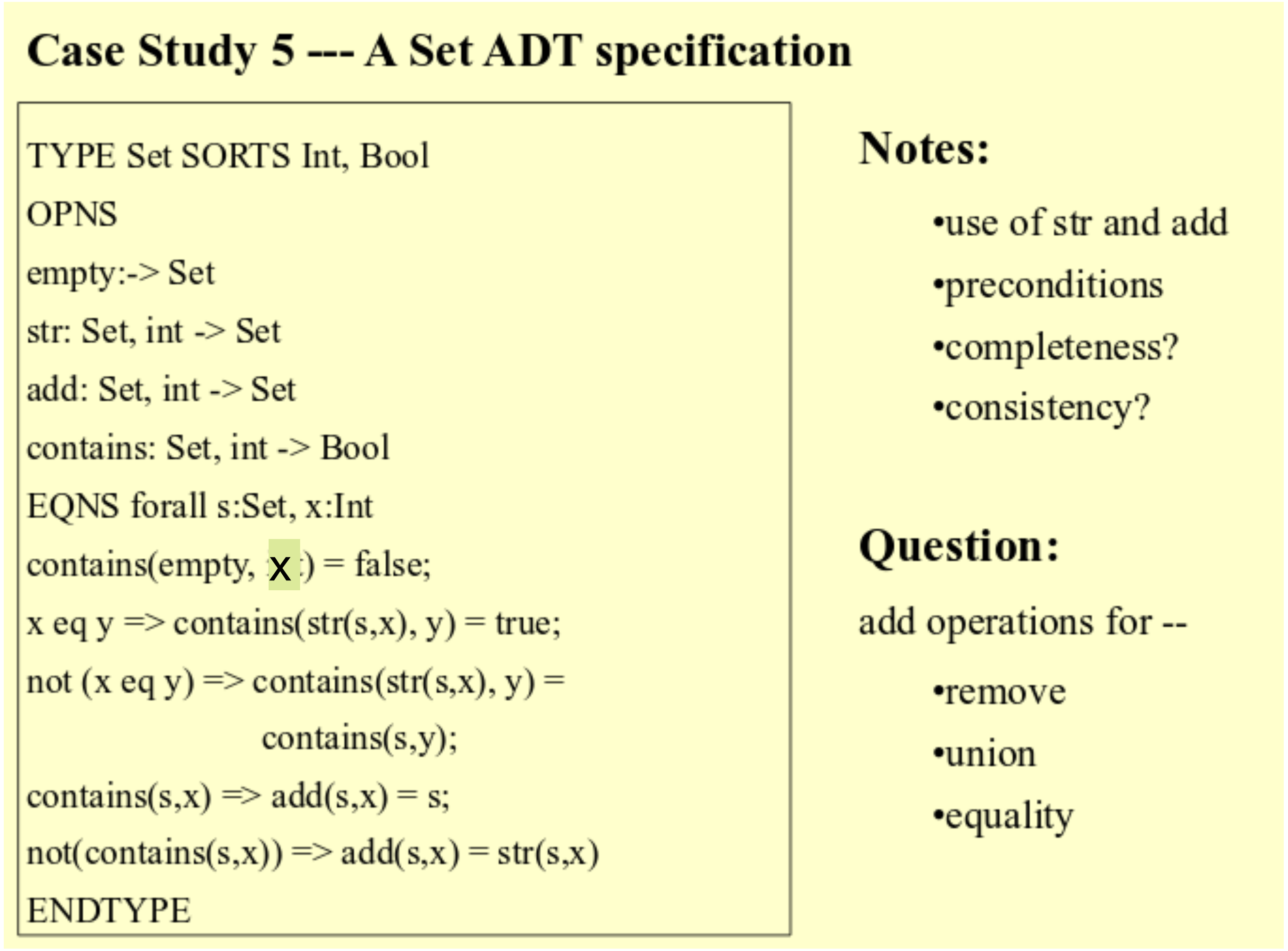
x eq x = true

Termination

x eq y = y eq x

Confluence

x eq x = false



Case Study 5 --- A Set ADT specification

```
TYPE Set SORTS Int, Bool
OPNS
empty:-> Set
str: Set, int -> Set
add: Set, int -> Set
contains: Set, int -> Bool
EQNS forall s:Set, x:Int
contains(empty, x) = false;
x eq y => contains(str(s,x), y) = true;
not (x eq y) => contains(str(s,x), y) =
                        contains(s,y);
contains(s,x) => add(s,x) = s;
not(contains(s,x)) => add(s,x) = str(s,x)
ENDTYPE
```

Notes:

- use of str and add
- preconditions
- completeness?
- consistency?

Question:

add operations for --

- remove
- union
- equality

Question:

Why do you think that ADTs are often used to specify OO models?

Problem -

Write an ADT specification for a stack of integers

Write an ADT specification for a queue of integers